

ASTU B.Ph. 2nd 30.05.14 (Reg)

Total No. of printed pages = 8

PY1322010

Roll No. of candidate

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2014

(Even Semester)

MATHEMATICS AND STATISTICS

Full Marks-100 Pass Marks-35 Time-Three hours

The figures in the margin indicate full marks for the questions.

1. For each of the following questions, four answers are provided of which one is correct. Choose the correct answer in each case. $1 \times 10 = 10$

- (i) The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^3 - 3\left(\frac{dy}{dx}\right)^4 = 1 \text{ is}$$

- (a) 4
(b) 3
(c) 2
(d) None of these.

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(ii) The solution of the differential equation $ydx - xdy = 0$ is

- (a) $y = cx$
- (b) $y = x + c$
- (c) $y = c$
- (d) None of these.

(iii) The Laplace transform of $2e^{3t}$ is

- (a) $\frac{2}{s-2}$
- (b) $\frac{2}{s-3}$
- (c) $\frac{2}{s+3}$
- (d) None of these.

(iv) The Laplace transform of $2 \cos 3t$ is

- (a) $\frac{6}{s^2 + 9}$
- (b) $\frac{2s}{s^2 + 4}$
- (c) $\frac{2s}{s^2 + 9}$
- (d) None of these.

(v) The inverse Laplace transform of $\frac{\sqrt{2}}{s^2+2}$ is

- (a) $\sin \sqrt{2}t$
- (b) $\sin 2t$
- (c) $\sin 4t$
- (d) None of these.

(vi) The probability of getting a double in two throws of an unbiased die is

- (a) $\frac{5}{36}$
- (b) $\frac{1}{6}$
- (c) $\frac{2}{36}$
- (d) None of these.

(vii) In Poisson distribution $P(X=r) = \frac{\lambda^r e^{-\lambda}}{r!}$,

$\lambda=np$ which of the following condition holds ?

- (a) $n \rightarrow 0, p \rightarrow \infty$
- (b) $n \rightarrow \frac{1}{2}, p \rightarrow \frac{1}{2}$
- (c) $n \rightarrow \infty, p \rightarrow 0$
- (d) None of these.

(viii) The value of the variable corresponding to maximum frequency is called

- (a) Mean
- (b) Mode
- (c) Variance
- (d) None of these.

(ix) For a platycurtic curve

- (a) $\beta_2 > 3$
- (b) $\beta_2 < 3$
- (c) $\beta_2 = 3$
- (d) None of these.

(x) For the coefficient of correlation

$$\rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sigma_x \sigma_y}, \text{ we have}$$

- (a) $-1 < \rho(X, Y) < 1$
- (b) $0 < \rho(X, Y) \leq 1$
- (c) $-1 \leq \rho(X, Y) \leq 1$
- (d) None of these.

2. Answer the following (any ten) : $4 \times 10 = 40$

(a) Solve : $(1 + y^2) dx + (1 + x^2) dy = 0$.

(b) $\frac{dy}{dx} = \frac{x^3 + y^3}{xy^2}$, solve the equation.

(c) Solve : $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 0$.

(d) Solve : $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{2x}$.

(e) Solve the differential equation :

$$\frac{d^2y}{dx^2} + 4y = \cos 3x.$$

(f) Solve the differential equation :

$$(D^2 - 4)y = \sin 2x, \quad D \equiv \frac{d}{dx}.$$

(g) There are two identical boxes containing respectively 5 white and 3 red balls, 8 white and 4 red balls. A box is chosen at random and a ball is drawn from it. If the ball drawn is white, what is the probability that the ball is drawn from the first box ?

(h) A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$. What is the probability that the problem is solved ?

(i) If the probability of a bad reaction from a certain injection is 0.002, determine the chance that out of 3000 individuals exactly 4 persons will get a bad reaction.

(j) In a binomial distribution of 10 trials the mean is given by $\mu = 5$. Find the probability of failure and hence find the variance of this distribution.

(k) Find the mean and median of the following data :

Class Interval :	0-10	10-20	20-30	30-40
Frequency :	5	10	4	1

(l) What do you mean by skewness and kurtosis ?

3. Answer the following questions. (any five) :
 $10 \times 5 = 50$

(a) Find the Laplace transform of

(i) $\sin^2 3t$

(ii) $\sin 2t \cos 3t$

(b) Find the inverse Laplace transforms of

(i) $\frac{2s-1}{(s-1)(s-2)}$

(ii) $\frac{3s}{s^2-2s-3}$

(c) Apply Convolution theorem to find

$$L^{-1}\left\{\frac{1}{s(s^2+4)}\right\}$$

(d) Prove that $L\{f'(t)\} = s\bar{f}(s) - f(0)$, provided $\lim_{t \rightarrow \infty} [e^{-st}f(t)] = 0$. Hence find $L\{f''(t)\}$.

(e) (i) In a survey of 500 students, the data obtained are as follows :

mean height = 155 cm and standard deviation = 15 cm. Assuming the heights to be normally distributed, find the number of students whose heights lie between 148 and 154 cm.

(ii) A random sample of size 16 has 53 as mean. The sum of squares of the deviation from mean is 135. Can this sample be regarded as taken from the population having 56 as mean ?

(f) Find the equation of the lines of regression based on the following data :

$$\sum x = \sum y = 15, \quad \sum x^2 = \sum y^2 = 49, \quad \sum xy = 44, \\ n = 5.$$

(g) Represent the following distribution by histogram and frequency polygon :

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	3	12	24	30	18

